Transition of patterns in liquid metal convection under a horizontal magnetic field

Takatoshi Yanagisawa[1], Yuji Tasaka[2], Ataru Sakuraba[3], Takehiro Miyagoshi[1] and Yozo Hamano[1]

[1] Department of Deep Earth Structure and Dynamics Research (D-EARTH), Japan Agency for Marine-Earth Science and Technology (JAMSTEC), Yokosuka 237-0061, Japan
[2] Laboratory for Flow Control, Division of Energy and Environmental Systems, Hokkaido University, Sapporo 060-8628, Japan
[3] Department of Earth and Planetary Science, University of Tokyo, Tokyo 113-0033, Japan

We performed both laboratory experiments and numerical simulations of Rayleigh-Bénard convection of an electrically conductive low-Prandtl-number fluid under a uniform horizontal magnetic field. The flow pattern is constrained, as the axes of convection rolls tend to align in the direction of the horizontal magnetic field. Ultrasonic measurement of flow velocity profile is suitable for this setting of liquid metal convection, because it can grasp quasi-two-dimensional structure with its time variations. Transitions of flow structure such as repetition of reversals of flow direction occur when the intensity of the magnetic field is in a limited range for a given Rayleigh number. By analyzing both the laboratory experiments and numerical simulations, we clarified the process of transitions as well as their mechanism. The process can be regarded as an interaction between aligned convection rolls and global-scale mean flow. The occurrence of global circulation bends the aligned rolls in a style of the skewed-varicose instability and induces roll number reduction. On the other hand, transitions can be regarded as a competition between two flow modes having different roll numbers and variations of their relative intensities in time.

Keywords: Rayleigh-Bénard convection, liquid metal, magnetic field, flow regimes

1. Introduction

To elcidate the way to turbulence is one of the most important topics in fluid dynamics. A typical setting to study transitions to turbulence is thermal convection in a layer driven by a vertical temperature gradient, that is, Rayleigh-Bénard convection (RBC). Many studies have been made on RBC (recent reviews in [1, 2]). Behaviors of thermal convection strongly depend on the Prandtl number (Pr) of the working fluid, and low Pr fluids such as liquid metals tend to be turbulent just after the onset of convection because of their low viscosity. On the other hand, liquid metals are ellectrically conductive fluids, and appling magnetic field to the system can controll the transition to turbulence. When a magnetic field is imposed on liquid metals, it enhances two-dimensionality and creates anisotropic flow structures with suppressing turbulence, depending on the direction and intensity of the field [3].

The controlling non-dimensional parameters in a RBC system under an imposed uniform magnetic field are the Rayleigh number (Ra), and the Chandrasekhar number (Q) [4]. In RBC of low Pr fluid like liquid metals, the region where two-dimensional (2D) rolls remain steady in the wave number-Rayleigh number space (the Busse balloon) is limited by the Eckhaus instability on the smaller wave number side, by the skewed-varicose instability on the larger wave number side, and by the oscillatory instability on the higher Ra values [5]. With a uniform horizontal magnetic field imposed, the Busse balloon extends by the increase of the onset Ra of oscillatory instability [6], and the values of Ra for further transitons to turbulece also increase. These are confiemed by earlier experiments [7,8]. The delays of transitons are convenient for the study of the way to turbulence, because the larger flow velocity at higher Ra makes it easier to observe the structure of the flow. It also means that transitions are controllable by the intensitiy of the magnetic field, while the value of Ra is fixed.

We conducted laboratory experiments on RBC of a liquid metal with observing the flow velocity by an ultrasonic velocity profiling method. We also performed a series of numerical simulations of RBC with electrically conductive low-Prandtl-number fluid under a uniform horizontal magnetic field. We reproduced flow structures and their time variations by numerical simulations that is consistent with observations in laboratory experiments. Based on these results, we can clearly grasp a route from a laminar flow with 2D rolls to a turbulent state.

2. Method

2.1 Laboratory Experiments

The vessel we used has a square geometry with aspect ratio five (Figure 1(a)) [9-11]. The horizontal scale of the vessel is 200 mm, and the thickness is 40 mm. The top and bottom plates are made of copper, and the temperature of each plate is maintained by circulating water. Liquid gallium is used as the working fluid. Transducers for the Ultrasonic Velocity Profiler are set in holes in the Teflon sidewalls, and are in direct contact with the liquid gallium. We are using UVP-Duo (Met-Flow S.A.), with the basic frequency of 4 MHz. The flow velocities of the gallium were measured along four lines from the transducers. The UVP measures the projected flow velocity along each line. We used a Helmholtz coil, or coils with large yoke to apply a uniform magnetic field (Figure 1(b)). The direction of magnetic field is horizontal in this study, and its intensity is controlled by an electric power supply. The maximum intensity of the magnetic field is 120 mT. The spatial variance of field intensity is within 2 % around the vessel. Refer [9] for more datails of the setting and method of these laboratory experiments.

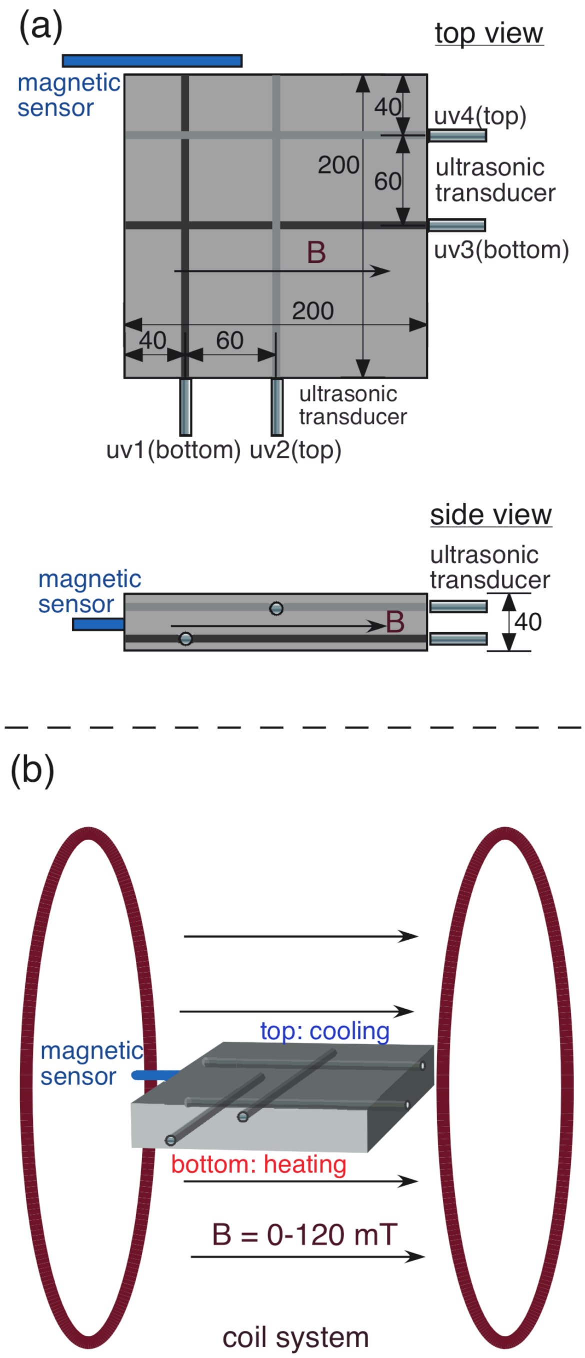


Figure 1: (a) Geometry of the vessel and ultrasonic beam lines for flow velocity measurement. Liquid gallium is filled in the vessel. The numbers are the dimensions in mm. (b) The vessel in a coil system. A uniform magnetic field **B** is applied in the horizontal direction.

2.2 Numerical Simulations

We performed numerical simulations for the same setting as laboratory experiments, with horizontal magnetic field imposed on the vessel of no-slip velocity boundaries. A set of magnetohydrodynamic equations are solved by a finite difference method with a uniform grid interval. We used realistic low value of Prandtl number for liquid metals (0.025). Refer [12] for the details and evaluation of the code.

3. Results

We identified several flow regimes depending on Ra and Q, between steady 2D laminar flow states and fluctuating large-scale flows with turbulence. We report here transitions on the decrease in Q, at a fixed Ra.

3.1 Time variations by Laboratory Experiments

Typical examples of the flow are shown in Figure 2, in a style of time-space map of horizontal flow velocity measured by the uv2 of Figure 1(a). All the panels show time variations for 2000 s. The intensity of the magnetic field, that is Q, is decreasing from top to bottom in this figure.

Figure 2(a) shows a laminar flow structure that has 5 rolls, with the roll axis paralell to the direction of the magnetic field. The flow parallel to the magnetic filed is very weak in this case. Very small amplitude of oscillation is observed. It comes from a periodic horizontal movement of the rolls, while the pattern keeps 2D state. This represents a typical flow regime under a strong horizontal magnetic field.

In Figure 2(b), the pattern keeps roll-like structure with dominant flow velocities perpendicular to the magnetic field, but repetitions of roll number transitions mainly 5–4 rolls are observed. Reversals of the flow direction in the rolls occur accompanied by the transitions.

Figure 2(c) is a state of oscillating convection, where the pattern shows large amplitude of oscillations with keeping a 4-roll state. The axes of rolls are winding in this oscillation. The period of oscillation is nearly equal to the circulation time of the flow for a roll.

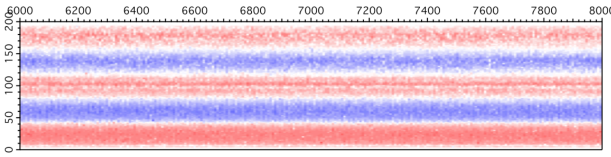
Figure 2(d) shows a state after a transition to turbulence occured, where whole- or half- vessel scale flows exist with short time fluctuations, and the magnitude of the velocitiy is similar in both directions perpendicular and parallel to the magnetic field. There are no roll-like structures, though the experiment is conducted with a horizontal magnetic field imposed. The behavior is similar to that without a magnetic field.

These results indicate that the number of rolls decreases with decrease in Q, from 5 to 1 or 2. The amplitude of spacial fluctuation increases in accordance with the decrease of the roll number, and continues to the fully developed turbulent state.

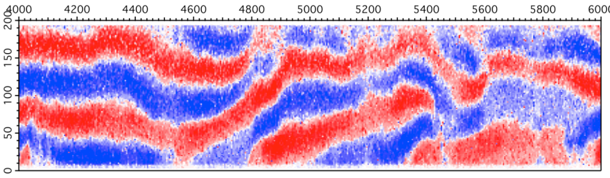
3.2 Flow structures by Numerical Simulations

In our numerical simulations, we first confirmed that both the time averaged number of rolls and their time variations are consistent with those observed in laboratory experiments. We present here 3D structures of flow in the whole vessel in Figure 3, by showing the isosurface of the Q_{3D}, that is the second invariant of the velocity gradient tensor. Though the values of the Chandrasekhar number Q are not exactly same as those in Figure 2, each of the regime of flow shown in Figure 3 corresponds to (a-d) of Figure 1.

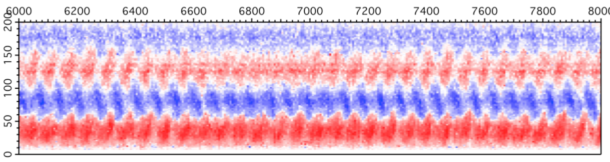
(a) Ra=3×10^4 and Q=8×10^3: five rolls, 2D oscillation



(b) Ra=3×10^4 and Q=1×10^3: roll number transitions



(c) Ra=3×10^4 and Q=7×10^2: four rolls, 3D oscillation



(d) Ra=3×10^4 and Q=8×10^1: no rolls, fluctuating cell

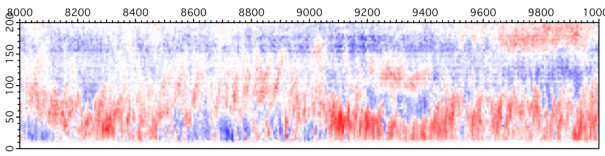


Figure 2: Typical examples of flow regimes. Time-space maps of the horizontal flow velocity are displayed for the measurement line at the center of the vessel, perpendicular to the applied magnetic field. The dulation of time is 2000 s. The direction and magnitude of the velocity are shown in the color scale, red is away from the transducer (located at distance=0) and blue is toward it.

Figure 3(a) is a laminar 5-roll structure with the roll axis fixed in the direction to the magnetic field. Small amplitude of horizontal oscillation of the rolls are also reproduced. As shown in the figure, in adition to the main 2D rolls, smaller secondary circulations exist between the rolls near the top or bottom boundary.

Figure 3(b) is a snapshot from a regime of roll number transition. A typical process of roll number transition is as follows. A 5-roll structure bends horizontally, and a roll at a sidewall is shrinking to make a 4-roll structure. Then, reconnection of the rolls between front and back occurs; Figure 3(b) shows just before the timing of a reconection. A new roll is growing along a sidewall, and 5-roll structure is reproduced. In this process, the key mechanism is an emergence of a global circulation in the horizontal plane. The horizontal circulation is related to the skewed-varicose instability of two-dimensional roll structure aligned in the direction of the magnetic field. This process of roll number transition can be thought as a repetition of two states those have different roll numbers.

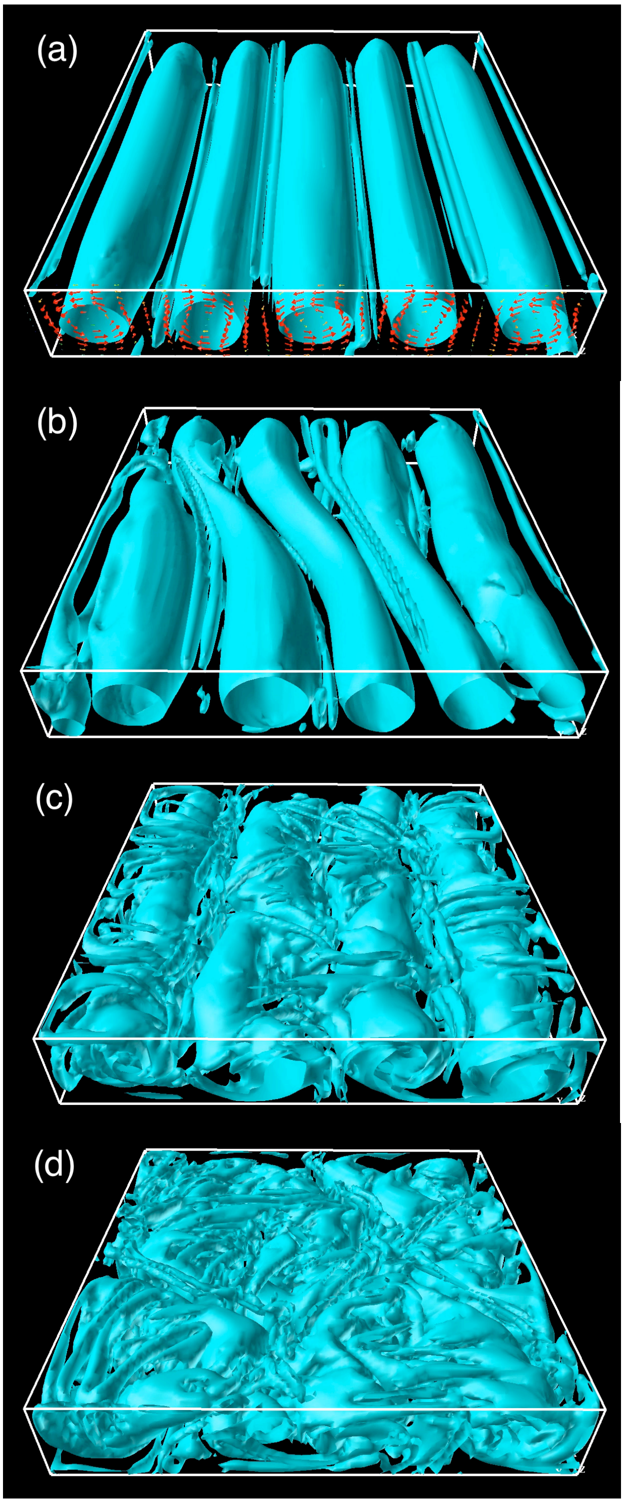


Figure 3: Typical examples of flow strucutres obtained by numerical simulations. The isosurfaces of Q_{3D}=0 are shown. A uniform magnetic field is applied from front to back; its intensity corresponds to (a) Q=6×10^3, (b) 3×10^3, (c) 7×10^2, and (d) 1×10^2. Ra is fixed as 3×10^4.

Figure 3(c) is a snapshot of 4-roll oscillation. The axes of rolls are winding and the amplitude of horizontal movement is large. A remarkable feature of this case is that many secondary vortices are accompanied by the oscillaiton of main rolls.

Finally, Figure 3(d) is a snapshot of a flow under a weak magnetic field. There are no distinct roll structure, but the flow shows large-scale isotropic features in average. Many secondary vortices are drifting with rise and fall on vessel-scale circulations.

4. Summary

Several transitions of flow pattern were identified in RBC of a liquid metal under a uniform horizontal magnetic field. In a decreasing way of the intensity of the magnetic field, these transitions are from (a) laminar 2D roll convection, to (b) roll number transitions, to (c) large amplitude of oscillatory rolls, and to (d) turbulent state with fluctuating large-scale circulations. In this process, the horizontal scale of the strucutre decreases with losing distinct rolls. The dominant flow in (a) is in the vertical plane perpendicular to the magnetic field (that is rolls), while the magnitude of circulations in the horizontal plane gets larger with decrease in Q. In the state (d), the power of horizontal circulations is as intese as that of vertical circulations.

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